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Thrust Performance of an Ideal Pulse Detonation Engine

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Quasi-steady and two-dimensional unsteady formulations of the problem on the operation cycle of a pulse detonation engine are derived. A formula for the specific impulse is obtained, and the thrust performance of the engine is calculated. It is found that the thrust performance of this engine for flight Mach numbers $\mathrm{M} \in [0; 3.6]$ and compression ratios $p_2/p_1 \in [1; 80]$ are always higher than those of the ramjet and one-spool turbojet. As the compression ratio increases, the advantage of the pulse detonation engine becomes less noticeable.

Key words: detonation, pulsed detonation engine, thrust performance.

INTRODUCTION

A pulse detonation engine (PDE) is a promising tool for motion in the air atmosphere with flight Mach numbers M = 0–3. Various schemes of such engines have been proposed [1–6]. In the simplest version, the PDE consists of the following units: inlet, which permanently takes in air and compresses it from the ambient atmospheric pressure p_1 to a certain stagnation value p_2; receiver, in which the air entering from the inlet is in a stagnant state with the pressure p_2; valve-based distribution system, which allows the air from the receiver to enter detonation chambers in a certain time sequence; set of detonation chambers, which is a system of identical cylindrical tubes with supersonic nozzles at the exit; fuel tank and a system for fuel injection into detonation chambers in accordance with a program correlated with air injection; system for detonation initiation.

In all detonation chambers, the same sequence of processes is cyclically repeated: filling of the chamber by compressed air with addition of fuel and formation of an explosive fuel–air mixture; detonation explosion of this mixture with the input valve closed, which is accompanied by a drastic increase in pressure; exhaustion of explosion products through the nozzle, which produces a reactive pulse. The phase shift of the processes in different detonation chambers allows for smaller oscillations of thrust and noise effects. Compression of air in the inlet is performed owing to dynamic pressure and/or a compressor. With some specifications, the PDE considered corresponds to schemes described in [4–6]. A preliminary analysis of PDE operation was performed in [7]. The present paper described the results of analytical and numerical investigations of the PDE thrust performance with two formulations of the problem of the PDE operation cycle.

FORMULATION OF THE PROBLEM

To determine the PDE thrust performance, we consider two mathematical models of the operation cycle under the following assumptions. Air and combustion products are assumed to be ideal gases with constant specific heats and a constant ratio of specific heats $\gamma = c_p/c_v$. Compression of air in the inlet and its motion in detonation chambers, as well as exhaustion (motion) of explosion products are isentropic processes without friction and heat exchange with the walls. The efficiency of the compressor (if present in the inlet) is 100%; the compressor is set into motion by an independent engine, which does not produce its own thrust and consumes the same fuel with a 100% efficiency. The explosion of the fuel–air mixture is simulated by an instantaneous release of energy Q per unit mass of air in

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the initial part of the chamber of length $L_0 \leqslant L$, where L is the total length of the detonation chamber.

Quasi-Steady Model (Model 1). We consider a quasi-steady formulation of the problem with a dimensionless parameter $\xi = L_0/L = 1$. The air is isentropically compressed in the inlet from the initial pressure p_1, density ρ_1, and velocity of sound c_1 to stagnation parameters p_2, ρ_2, and c_2:

$$\frac{p_2}{p_1} = \pi, \quad \frac{\rho_2}{\rho_1} = \pi^{1/\gamma}, \quad \frac{c_2^2}{c_1^2} = \pi^{1-1/\gamma}. \qquad (1)$$

It was additionally assumed that the flow in the chamber and in the nozzle is one-dimensional and is divided into three stages. At the initial stage, $t = 0$, after instantaneous energy release with the thermal effect Q per unit mass of the mixture, the pressure in the detonation chamber also becomes instantaneously identical and takes the value p_3 corresponding to conservation of entropy of explosion products and total internal energy of the gases in the chamber:

$$\rho_3 = \rho_2, \quad \frac{p_3}{p_2} = \frac{c_3^2}{c_2^2} = 1 + \gamma(\gamma - 1)\frac{Q}{c_2^2}. \qquad (2)$$

At stage I ($0 < t \leqslant t_1$), there occurs quasi-steady exhaustion of gases through the nozzle with a velocity of sound $c(t)$, density $\rho(t)$, and pressure in the chamber $p(t)$ gradually decreasing from p_3 to p_2. During exhaustion, the variation of the gas mass in the chamber m in time satisfies the ordinary differential equation

$$\frac{dm}{dt} = S_c L \frac{d\rho(t)}{dt} = -B\rho(t)c(t)S_{\min}.$$

Here, S_c is the cross-sectional area of the detonation chamber, $S_{\min}$ is the throat area, and

$$B = \Big(\frac{2}{\gamma + 1}\Big)^{0.5(\gamma+1)/(\gamma-1)}.$$

Integrating this equation, we find the explicit dependence for the sought functions at stage I:

$$\frac{c(t)}{c_3} = (1 + at)^{-1}, \quad \frac{\rho(t)}{\rho_3} = \Big[\frac{c(t)}{c_3}\Big]^{2/(\gamma-1)},$$

$$\frac{p(t)}{p_3} = \Big[\frac{c(t)}{c_3}\Big]^{2\gamma/(\gamma-1)}, \qquad (3)$$

$$\frac{p(t)}{p_4(t)} = \frac{p(t_1)}{p_4(t_1)} = \frac{p_2}{p_1} = \pi,$$

$$t \leqslant t_1 = \frac{1}{a}\Big[\Big(\frac{p_3}{p_2}\Big)^{0.5(1-1/\gamma)} - 1\Big],$$

$$a = \frac{\gamma - 1}{2} B \frac{c_3 S_{\min}}{L S_c}.$$

Here $p_4(t)$ is the pressure at the nozzle exit and t_1 is the duration of stage I.

Then, exhaustion passes to the steady stage II ($t_1 < t \leqslant t_1 + t_2$) where the remaining products are displaced from the chamber by the next portion of air and the fuel–air mixture at a constant pressure p_2 and a constant velocity of gases inside the chamber $u_c \ll c_c$:

$$p(t) = p_2, \quad \rho(t) = \rho_3 (p_2/p_3)^{1/\gamma}, \qquad (4)$$

$$0 < t - t_1 \leqslant t_2 = \frac{\gamma - 1}{2a}\Big(\frac{p_3}{p_2}\Big)^{0.5(1-1/\gamma)}.$$

The throat area $S_{\min}$ and the nozzle-exit area S_4 are chosen such that the pressure p_4 at the nozzle exit at stage II is equal to the ambient atmospheric pressure p_1 and the inequality $u_c \ll c_c$ is satisfied:

$$\frac{S_{\min}}{S_c} = \Big(\frac{c_2}{c_3}\Big)^{1/\gamma} \frac{u_c}{B\, c_1} \Big/ \pi^{(\gamma-1)/2\gamma},$$

$$\frac{S_4}{S_{\min}} = B\Big[\frac{(\gamma - 1)\pi^{1+1/\gamma}}{2(\pi^{1-1/\gamma} - 1)}\Big]^{1/2}.$$

In accordance with the law of conservation of momentum, the current impulse I at each time t is

$$I(t) = S_4[\rho_4(t)u_4^2(t) + p_4(t) - p_1] - Gu_1.$$

Here $G = \rho_1 u_1 S_1$ is the mass flow rate of air, u_1 is the gas velocity at the entrance, and S_1 is the cross-sectional area of the engine entrance. Then, the impulse I_s averaged over the period $t = t_1 + t_2$ is found from the relation

$$I_s = \frac{1}{t}\int_0^t I(t)dt, \quad J = \frac{I_s}{G}. \qquad (5)$$

Substituting dependences (1)–(4) into Eq. (5) and integrating the latter, we find the PDE thrust performance from the following system of algebraic equations:

$$\frac{J}{c_1} = \frac{2}{\gamma + 1}\Big(\mathrm{M}_4 + \frac{1}{\gamma \mathrm{M}_4}\Big)\Big[\frac{c_3}{c_2} - \Big(\frac{c_2}{c_3}\Big)^{1/\gamma}\Big] + \Big(\frac{c_2}{c_3}\Big)^{1/\gamma}\Big(\mathrm{M}_4 - \frac{t_1}{\gamma \mathrm{M}_4 t_2}\Big) - M_1,$$

$$\mathrm{M}_4^2 = \frac{2}{\gamma - 1}(\pi^{1-1/\gamma} - 1) + \frac{u_c^2}{c_4^2}, \quad c_4^2 = c_3^2\Big(\frac{p_1}{p_3}\Big)^{1-1/\gamma},$$

$$\nu = (t_1 + t_2)^{-1}, \quad \frac{F}{S_c p_1} = \frac{\nu J \rho_2 L}{p_1}, \qquad (6)$$

$$\frac{c_1^2 \mathrm{M}_1^2}{2} + \frac{c_1^2}{\gamma - 1} + \frac{N}{G} = \frac{\gamma + 1}{2(\gamma - 1)} c_2^2.$$

Here $\mathrm{M} = u/c$ is the Mach number, N is the compressor power, ν is the number of cycles of chamber operation, J is the mean specific impulse, and $F/S_c p_1$ is the dimensionless thrust normalized to the unit cross-sectional

TABLE 1

Dimensionless Specific Impulse and Thrust of Ideal Jet-Propulsion Engines

p_2/p_1	p_3/p_1	M_4	ν, Hz	J/c_1 for $M_1 \ll 1$		J/c_1 for $M_1 = M_4$		$F/S_c p_1$ (PDE)	
				PDE	Turbojet	PDE	Scramjet	$M_1 \ll 1$	$M_1 = M_4$
2	19.2	1.05	42 (40)	3.87 (3.68)	2.80	2.82	1.76	1.24 (0.74)	0.905
3	26	1.36	43 (43.5)	4.20 (4.03)	3.46	2.84	2.10	1.85 (1.17)	1.25
4	32.2	1.56	43.6 (44)	4.44 (4.29)	3.83	2.88	2.27	2.43 (1.56)	1.58
6	43.8	1.83	44.4 (46)	4.75 (4.61)	4.29	2.92	2.46	3.54 (2.33)	2.18
10	64.4	2.16	46 (48)	5.11 (4.98)	4.77	2.95	2.61	5.72 (3.80)	3.30
20	109	2.60	48 (52)	5.55 (5.48)	5.32	2.95	2.72	10.66 (7.33)	5.66
40	186	3.06	50 (55)	5.97 (6.03)	5.81	2.91	2.75	19.5 (14.1)	9.50
80	320	3.54	52 (59)	6.38 (6.67)	6.27	2.84	2.73	35.9 (27.6)	16.0

Note. The numbers in brackets refer to calculations by model 2.

area of the detonation chamber. The subscripts refer to parameters of ambient air (1), parameters of the stagnant air in the receiver (2), parameters in the chamber at the beginning of stage I (3), flow parameters at the nozzle exit (4), and parameters in the chamber at stage II (c).

Two-Dimensional Unsteady Model (Model 2). To verify the assumptions in model 1, the problem on thrust performance of an ideal PDE was formulated and solved numerically in a two-dimensional unsteady formulation. We considered a detonation chamber, which was a cylindrical tube of length L and radius r_c, with a supersonic nozzle at the exit (the nozzle-throat radius was $r_{\min}$). The ambient medium was air with a pressure p_1 and temperature T_1. At the initial time $t = 0$, the whole tube (or part of its $0 < x < L_0$) was filled by a fuel–air mixture with a pressure p_2 and temperature T_2. The valve at the tube entrance ($x = 0$) was closed. There occurred an instantaneous explosion of the mixture with a thermal effect Q per unit mass of the mixture and subsequent exhaustion of gases through the nozzle. At the time $t = t_1$, when the pressure at the left end of the tube became equal to the pressure in the receiver p_2, the valve was opened, and the remaining explosion products were displaced from the chamber by the next portion of air and the fuel–air mixture. At the time $t = t_1 + t_2$, the interface between the fresh mixture and explosion products reached the coordinate $x = L$, the valve was closed, and another instantaneous explosion of the mixture occurred in the cylindrical tube (or in its part $0 < x < L_0$). The cycle was repeated again and again. We had to determine the dynamics of the exhaustion process and the PDE reactive impulse during the period $\Delta t = t_1 + t_2$. The behavior of the gas in the chamber was described by unsteady equations of gas dynamics

$$(\rho r)_t + (\rho u r)_x + (\rho v r)_r = 0,$$

$$(\rho u r)_t + [(\rho u^2 + p)r]_x + (\rho u v r)_r = 0,$$

$$(\rho v r)_t + (\rho u v r)_x + [(\rho v^2 + p)r]_r = p,$$

$$(E r)_t + [(E + p)u r]_x + [(E + p)v r]_r = 0,$$

where ρ is the density, p is the pressure, u and v are the components of the velocity vector, $E = \rho[e + (u^2 + v^2)/2]$, and $e = p/(\gamma - 1)\rho$. The behavior of the gas outside the chamber was calculated in a limited region whose volume was approximately 10^3 of the chamber volume. On the outer side boundary of the computational domain, i.e., for $r = R$ ($R \approx$ 10–15r_c), the gas parameters were assumed to be equal to the parameters of the undisturbed atmosphere. All variables were normalized to the initial parameters with an appropriate dimensionality: p_1, ρ_1, L, $\sqrt{p_1/\rho_1}$, and $L/\sqrt{p_1/\rho_1}$. The formulated problem on PDE operation is characterized by seven dimensionless criteria of similarity

$$\pi = p_2/p_1, \quad \theta = T_2/T_1, \quad \xi = L_0/L,$$

$$q = Q/c_1^2, \quad \gamma, \quad \delta = r_{\min}/r_c, \quad r_c/L$$

determined from the initial stagnation parameters, characteristics of the fuel–air mixture, and chamber geometry. To have a correct comparison between the solution of the unsteady problem and the quasi-steady approximation, the numerical values of the first six criteria were the same as those in calculations by model 1. The value of the parameter $r_c/L = 1/12$ was fixed.

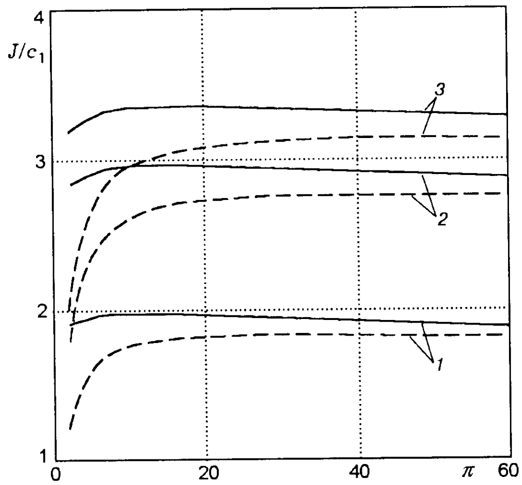

Fig. 1. Dimensionless specific impulse of the PDE (solid curves) and ramjet (dashed curves) versus the compression ratio: Q = 1 (1), 1.6875 (2), and 2 MJ/kg (3).

CALCULATION RESULTS

Model 1. In the calculations, we assumed that $\gamma = 1.4$, $c_1 = 300$ m/sec, $Q/c_1^2 = 18.75$, and $u_c = c_1/3$. The compression ratio π was defined and varied. The flight Mach number varied from zero to $M_1 = M_4$, corresponding to compression of air to the pressure p_2 in an ideal inlet of a ramjet without the compressor. Some results of calculations by formulas (6) are listed in Table 1. It follows from the analysis of the dependences of the specific impulse J/c_1 of the PDE ($M_1 = M_4$) on the compression ratio π for fixed values of energy Q (Fig. 1) that the values of J/c_1 vary nonmonotonically with increasing π. In the interval $10 < \pi < 20$, the specific impulse of the PDE reaches a maximum. The resultant PDE thrust performance is compared to the ramjet parameters [8]

$$J/c_1 = M_4\sqrt{1 + (\gamma - 1)Q/c_1^2/\pi^{1-1/\gamma}} - M_1,$$

which were calculated under similar assumptions and for the same values of π. It was assumed that combustion in the ramjet occurs without total pressure losses. The calculations for $M_1 \in [0; 3.6]$ and $\pi \in [1; 80]$ showed that the PDE thrust performance is always better than that of the ramjet and one-spool turbojet. In terms of specific impulse, the ideal PDE is inferior only to an ideal bypass turbojet with a high bypass ratio; nevertheless, the PDE exceeds the latter in terms of head thrust (see the last column in Table 1). At the same time, the PDE advantages become less pronounced with increasing compression ratio.

Model 2. In solving numerically the two-dimensional unsteady problem, we obtained the distributions of gas-dynamic parameters in the chamber at different times t, and the time evolution of the pressure $p(t)$ at the tube entrance ($x = 0$), mass flux

$$G(t) = \int_S \rho u ds,$$

and momentum flux

$$I(t) = \int_S (p + \rho u^2 - p_1)ds$$

in the nozzle-exit section.

Some results are presented in Table 1. Typical dependences of pressure over three periods of oscillations $P = p/p_1$ at the detonation-chamber entrance on the dimensionless time $\tau = t\sqrt{p_1/\rho_1}/L$ are plotted in Fig. 2b, and the dynamics of the instantaneous specific impulse $J_i(t) = I(t)/G(t)$ and the mean specific impulse

$$J(t) = \int_0^t I(t)dt \Big/ \int_0^t G(t)dt$$

in the nozzle-exit section of the PDE is shown in Fig. 3. After the first "nonstandard" oscillation with a period Δt_1, the solution of the two-dimensional unsteady problem reaches a periodic regime with a constant period $\Delta t < \Delta t_1$ and specific impulse $J < J_1$. The solution of the problem for the first period corresponds to the single-cycle mode in a motionless ambient medium with the pressure p_1 and temperature T_1 [9, 10]. Note, unsteadiness of the process of PDE operation leads to significant differences in instantaneous and mean thrust characteristics of the PDE during each period of oscillations. According to the calculated data from Fig. 3, the instantaneous specific impulse $J_i(t)$ changes by a factor of 2.6 during one period, whereas the mean specific impulse $J(t)$ changes only by 30%. Calculations by model 2 (see Table 1) predict a decrease in specific impulse (per unit mass of the fuel–air mixture) by no more than 5%, as compared to model 1, due to unsteadiness and flow nonuniformity. Apparently, these losses can be reduced by nozzle optimization. The decrease in thrust is more significant: its reaches 40%. It is caused by the decrease in pressure, density, and hence, mass of the gases that fill the chamber in each cycle with an unchanged frequency of explosions. The decrease in the mean pressure in the filled chamber is confirmed by a comparison of the dependences $p(t)$ (see Fig. 2) obtained in two different formulations of the problem with identical initial data.

The calculations of PDE operation showed that the PDE specific impulse increases (Fig. 4) and the thrust

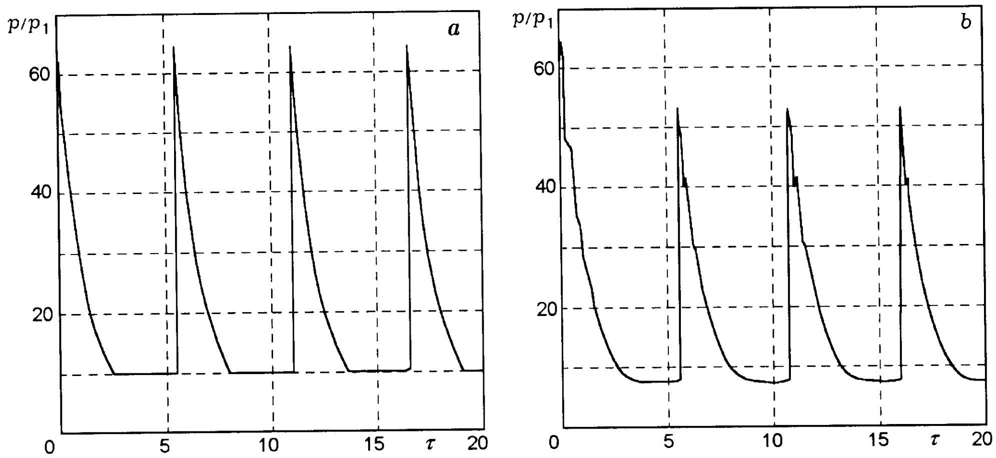

Fig. 2. Dimensionless pressure at the entrance to the detonation chamber versus dimensionless time: $\pi = 10$, $\xi = 1$, $\delta = 0.46$; the calculations were performed by model 1 (a) and model 2 (b).

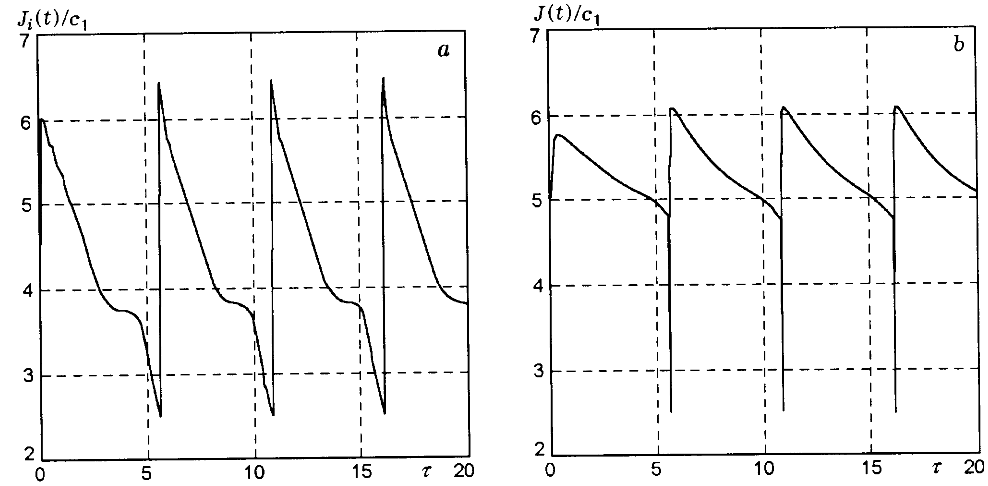

Fig. 3. Instantaneous specific impulse (a) and mean specific impulse (b) in the nozzle-exit section versus dimensionless time ($\pi = 10$, $\xi = 1$, and $\delta = 0.46$).

and frequency of cycles decrease as the degree of filling of the chamber by the fuel–air mixture decreases (the parameter ξ decreases), similar to the single-cycle regime [9]. For $\xi \in (1/4, 1/3)$, $\pi \in (4, 40)$, and $M_1 \ll 1$, the specific impulse averaged over the period was $J \approx 3000$ m/sec, which corresponds to the specific flow rate of the hydrocarbon fuel (propane) approximately equal to 0.12 (kg/h)/N. Note, if there is no nozzle ($S_{\min} = S_c$), the unsteady calculation performed for the compression ratio $\pi = 10$ predicts a decrease in the specific impulse J/c_1 (dashed curve in Fig. 4) by 18%. This means that the non-contoured detonation chamber in the form of a cylindrical tube is not optimal from the viewpoint of obtaining the highest specific impulse of the PDE.

CONCLUSIONS

By means of calculations by two models, it is established that the thrust performance of an idealized PDE with $M \in [0; 3.6]$ and $p_2/p_1 \in [1; 80]$ is always better than that of an idealized ramjet and one-spool turbojet. The use of non-contoured detonation cham-

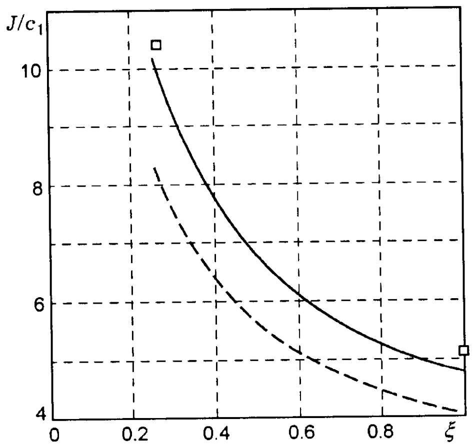

Fig. 4. Dimensionless specific impulse of the PDE versus the degree of filling of the chamber by the fuel–air mixture with $\pi = 10$: the points refer to model 1; the solid and dashed curves refer to model 2 with $\delta = 0.46$ and $\delta = 1$, respectively.

bers in the PDE decreases the specific impulse. A comparison of results calculated by models 1 and 2 allows us to state that the algebraic formulas of the quasi-steady model offer an accurate estimate (within 5%) of the PDE specific impulse. Therefore, for engineering calculations of the thrust performance of an ideal PDE, one can use Eqs. (6), thus, avoiding time-consuming two-dimensional unsteady calculations. Application of model 2 is reasonable at the final stage of the analysis of PDE operation efficiency for refining PDE thrust characteristics in the vicinity of the region of optimal parameters.

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