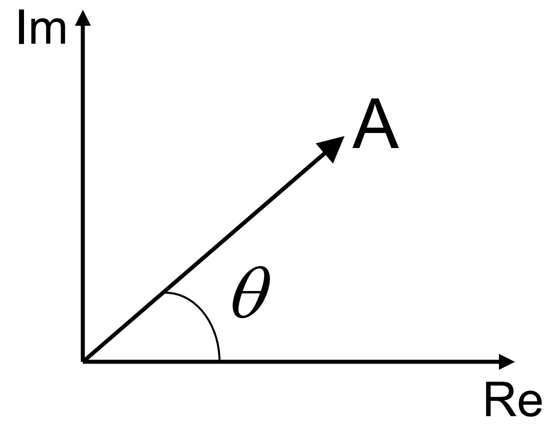
ROOT LOCUS (RL)

- Graphical technique that will allow us to plot the path or locus of the eigen values as one parameter changes (0 to ∞)

- RL has the ability to
  - Determine closed loop system behavior from open loop conditions
    - Determine the effects of one parameter qualitatively

Complex Algebra Review



$$A = \alpha + \beta j = |A| e^{j\theta}$$

$$\theta = \operatorname{atan2}\left(\frac{\beta}{\alpha}\right) = \angle A$$

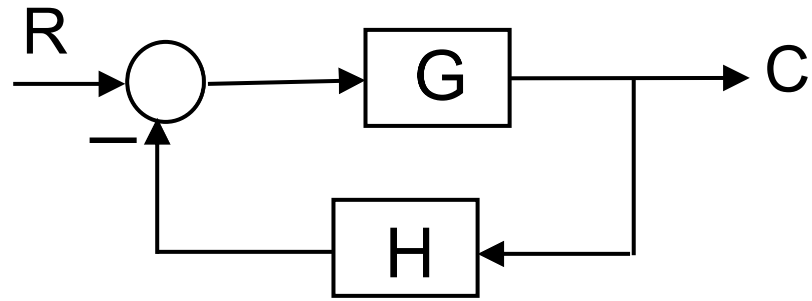
$$|A| = \sqrt{\alpha^2 + \beta^2}$$

$$\frac{A_1}{A_2} = \frac{|A_1| e^{j\theta_1}}{|A_2| e^{j\theta_2}} = \frac{A_1}{A_2} e^{j(\theta_1 \pm \theta_2)}$$

$$A_1 \pm A_2 = (\alpha_1 + \beta_1 j) \pm (\alpha_2 + \beta_2 j)$$
$$= (\alpha_1 \pm \alpha_2) + (\beta_1 \pm \beta_2) j$$

Background on RL

CLTF $$\frac{C}{R}=\frac{G}{1+GH}$$



$$CE:\ 1+GH=0$$

GH: Ratio of polynomials in s
Where s: complex quantity

$$s=\sigma\pm j\omega$$

$$1+GH\big|_{s=\sigma\pm j\omega}=0$$

$$GH\big|_{s=\sigma\pm j\omega}=-1$$

Magnitude $\left| GH\big|_{s=\sigma\pm j\omega} \right| = |-1|$

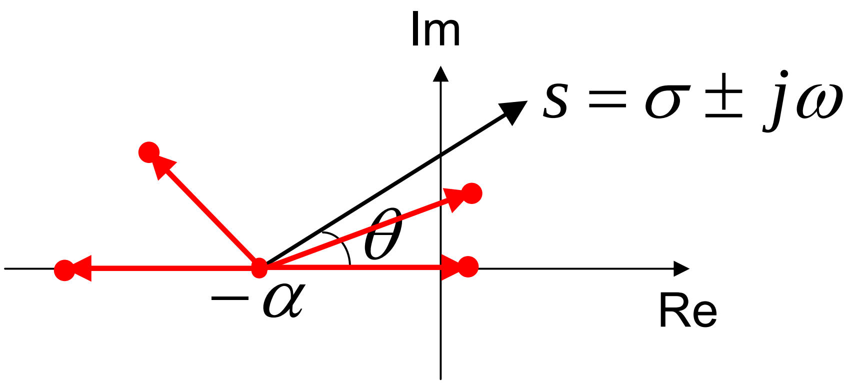
Angle $\angle GH\big|_{s=\sigma\pm j\omega} = \angle(-1+0j)$

$$= \pm 180(2n+1) \quad n = 0,1,2\ldots$$

EXAMPLE

$$GH = \frac{k}{s+\alpha}$$

$$GH = \frac{k}{(\sigma+\alpha)+j\omega}$$

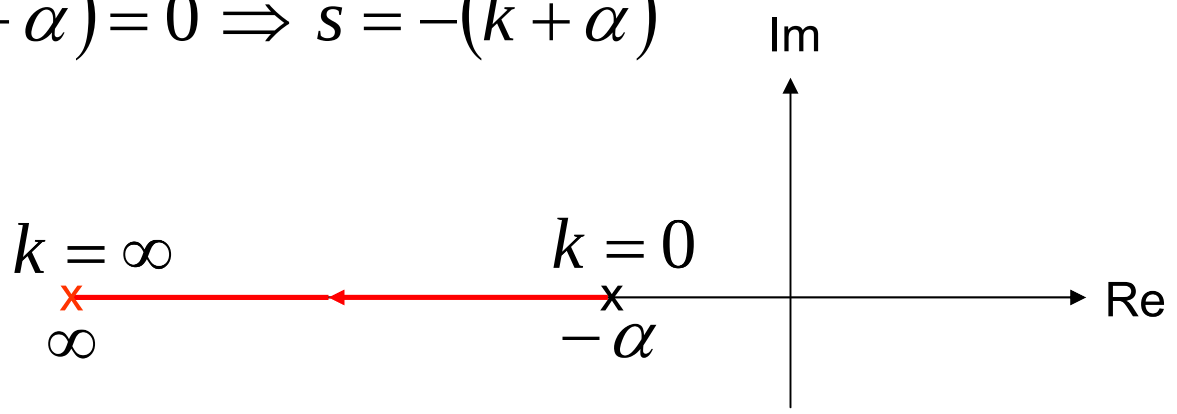


$$|GH| = \frac{k}{\sqrt{(\sigma+\alpha)^2 + \omega^2}}$$

$$\angle GH = \theta = -\text{atan}\frac{\omega}{\sigma+\alpha} = \pm 180(2n+1)$$

$$GH + 1 = 0 \;\Rightarrow\; \frac{k}{s+\alpha} + 1 = 0$$

$$s + (k+\alpha) = 0 \Rightarrow s = -(k+\alpha)$$



Properties of RL

$$GH = -1 \quad \text{where} \quad G = k_g \frac{N_g}{D_g} \;\; \& \;\; H = k_f \frac{N_f}{D_f}$$

$$k_g \frac{N_g}{D_g} k_f \frac{N_f}{D_f} = -1 \;\Rightarrow\; k = k_g k_f = -\frac{N_g N_f}{D_g D_f}$$

$$\text{if} \quad k = 0 \;\Rightarrow\; \left.\begin{aligned} D_g D_f &= 0 \\ N_g N_f &= \infty \end{aligned}\right\} \text{Open loop poles}$$

$$\text{if} \quad k = \infty \;\Rightarrow\; \left.\begin{aligned} D_g D_f &= \infty \\ N_g N_f &= \infty \end{aligned}\right\} \text{Open loop zeros}$$

$$\text{OLTF}: \quad k\frac{N_g N_f}{D_g D_f}$$

$$\text{CLTF}: \quad \frac{C}{R} = \frac{k_g \dfrac{N_g}{D_g}}{1 + k\dfrac{N_g N_f}{D_g D_f}} = \frac{k_g \dfrac{N_g}{D_g}}{\dfrac{\left(D_g D_f + kN_g N_f\right)}{D_g D_f}}$$

$k \to 0$: CL poles approaches OL poles

$k \to \infty$: CL poles approaches OL zeros

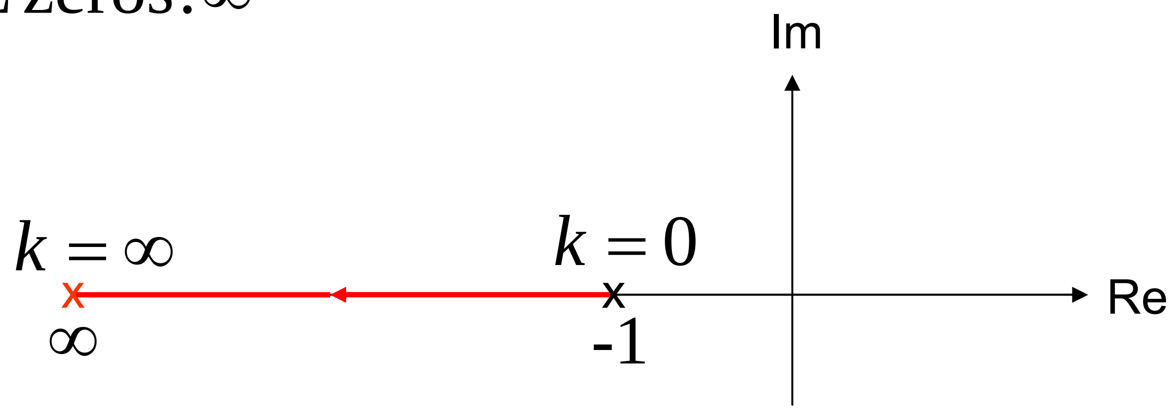
EXAMPLE

$$GH = \frac{k}{s+1} \qquad 0 \le k \le \infty$$

$$CE: s + (1+k) = 0$$

$$\text{OL poles}: \quad s = -1$$

$$\text{OL zeros}: \infty$$

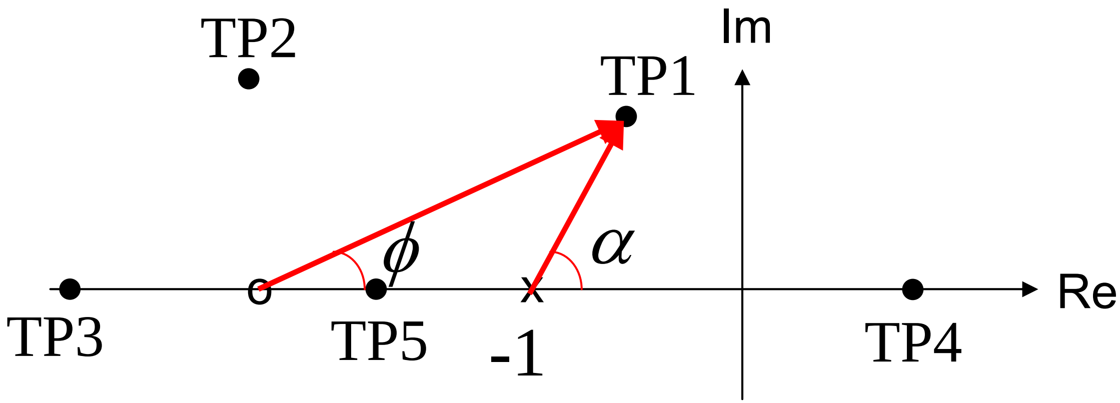


EXAMPLE

$$GH = \frac{k(0.4s+1)}{s+1}$$

$$\text{OL pole}: \; s = -1$$

$$\text{OL zero}: \; s = -2.5$$



$$\angle GH = \angle k + \angle 0.4s + 1 - \angle s + 1$$

$$\angle GH = 0 + \phi - \alpha = \pm 180(2n+1)$$

$$CE: (1+0.4k)s+1+k=0$$

$$s=-\frac{1+k}{1+0.4k}$$

$$k=0 \Rightarrow s=-1$$

$$k=\infty \Rightarrow s=-2.5$$

$$k=5 \Rightarrow s=-2$$

$$k=100 \Rightarrow s=-\frac{101}{41}$$

$$k=0 \Rightarrow s=-\frac{1001}{401}$$

$$\left|GH\right|_{s=-1.5} = \left|1\right|$$

$$\left|\frac{k(0.4s+1)}{s+1}\right|_{s=-1.5} = 1$$

$$\left|-0.8k\right| = 1$$

$$k = 1.25$$