ROOT LOCUS (Cont…)

Starts at O.L. poles k varies from 0 to ∞

Ends at O.L. zeros.

$1 + kGH = 0$ where kGH is OLTF

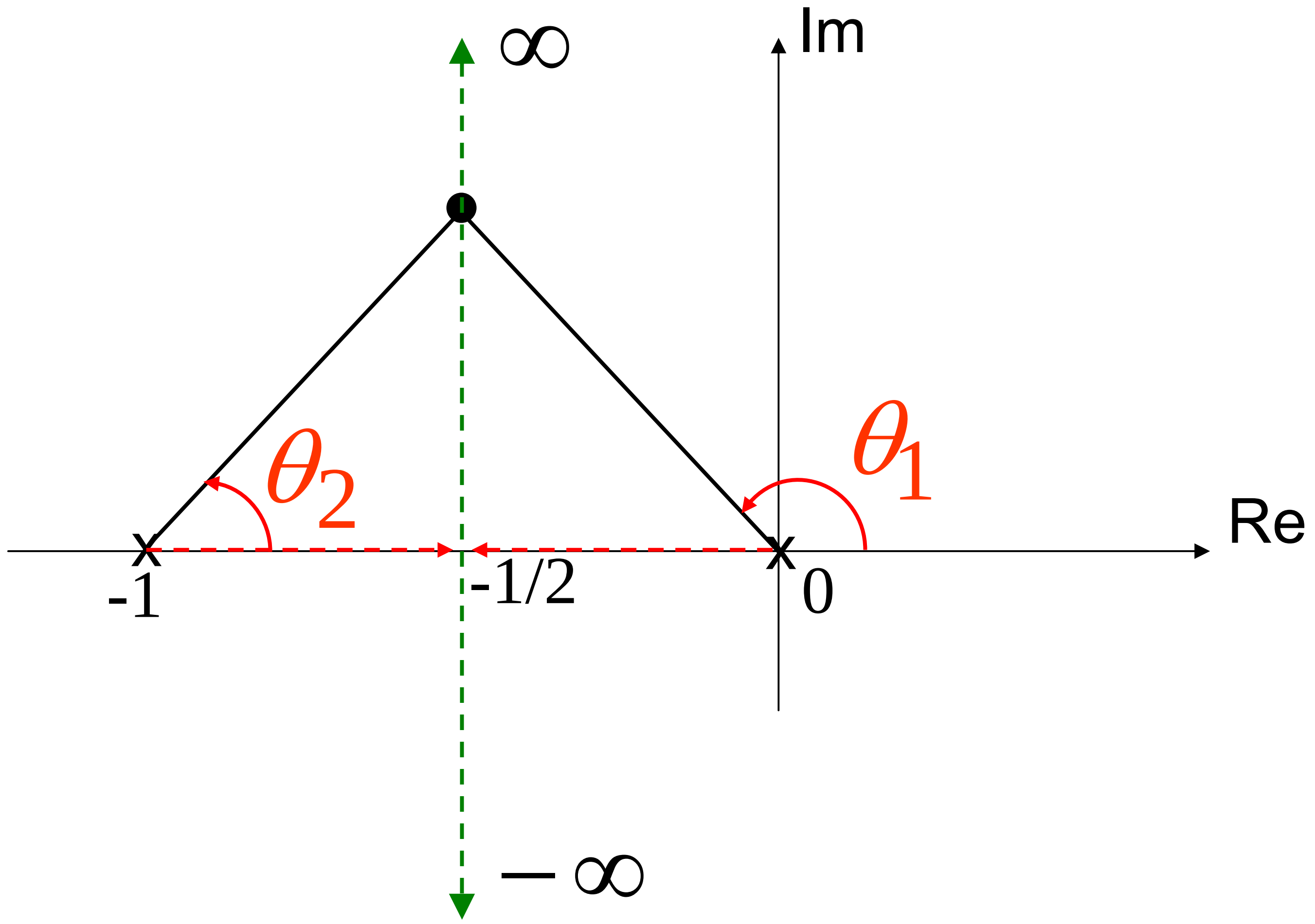
OLTF: $GH = \dfrac{k}{s(s+1)}$

OL Poles: $s = 0$, $s = -1$

OL Zeros: none

$$CE: \left(s^2 + s\right) + k = 0$$

$$s_{1,2} = \frac{-1 \pm \sqrt{1-4k}}{2} = -\frac{1}{2} \pm \frac{1}{2}\sqrt{1-4k}$$



∞
Im
θ1
θ2
Re
-1
-1/2
0
− ∞

k	$CL\ poles$	
0	0	-1
$\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{2}$
$\frac{1}{2}$	$-\frac{1}{2}+\frac{1}{2}j$	$-\frac{1}{2}-\frac{1}{2}j$
10	$-\frac{1}{2}+\frac{\sqrt{39}}{2}j$	$-\frac{1}{2}-\frac{\sqrt{39}}{2}j$
1000	$-\frac{1}{2}+\frac{\sqrt{3999}}{2}j$	$-\frac{1}{2}-\frac{\sqrt{3999}}{2}j$

Identify controller gain k to yield eigen value at

$$s=-\frac{1}{2}\pm 3j$$

$$\left|GH\right|_{\text{for all } s}=\left|-1\right|=1$$

$$\left|\frac{k}{s(s+1)}\right|_{s=-\frac{1}{2}+3j}=1$$

find k.

$$0 < k < \frac{1}{4}, \text{the eigen values are on the the Re-axis}$$

$$\text{i.e. they behave as the first order system.}$$

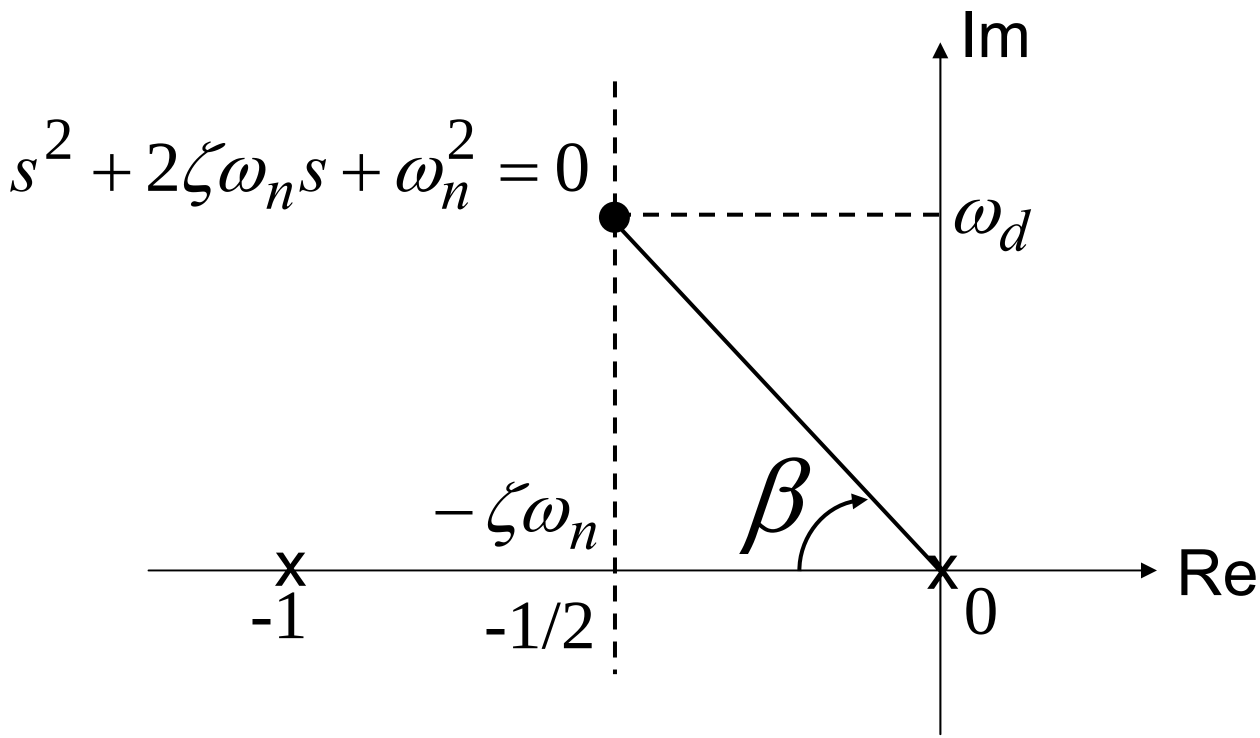
$$(s+\alpha)(s+\beta), \zeta > 1.$$

$$k = \frac{1}{4}, \text{the eigen values are on the Re-axis}$$

$$\text{\& repeating. } (s+\gamma)^2, \zeta = 1.$$

$$k > \frac{1}{4}, \text{the eigen values are complex conjugates.}$$

$$\zeta < 1.$$



$$\tan\beta = \frac{\omega_d}{\zeta\omega_n} = \frac{\omega_n\sqrt{1-\zeta^2}}{\omega_n\zeta} = \left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

EXAMPLE

$\zeta = 0.6$ (desired damping)

$$\tan\beta = \frac{\sqrt{1-0.6^2}}{0.6} = 1.333 \Rightarrow \beta = 53.1^o$$

$$-\zeta\omega_n = -\frac{1}{2} \Rightarrow \omega_n = 0.833$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 0.6664$$

Point of interest $\Rightarrow$ Desired eigen value

Find k to yield $s = -\frac{1}{2} \pm 0.6664j$

RULES FOR DRAWING ROOT LOCUS

Example

$$\text{OLTF}: \quad GH = \frac{k}{(s+1)(s+2)(s+3)}$$

- Find OL poles : 3 $(-1, -2, -3)$

  OL zeros : 0

- Draw OL poles & zeros

- Start OL poles $(k = 0)$

  End OL zeros $(k \to \infty)$

- # Loci = 3

- Symmetry about $\text{Re}-\text{axis}$
- $n \rightarrow \# OL \text{ poles} = 3$

  $m \rightarrow \# OL \text{ zeros} = 0$
- $\#\text{asymptotes} = n_a = n - m = 3$

  (loci going to ∞)
- Angle of asymptotes with the $\text{Re}-\text{axis}$

$$\theta_q = \frac{\pm 180(2q+1)}{n-m}$$

$$= \frac{\pm 180(2q+1)}{3} = \pm 60(2q+1)$$

q	θ_q
0	± 60
1	± 180
2	± 300

- Intersection of asymptotes with Re – axis

(Centroid of poles & zeros)

$$\sigma_a = \frac{\sum \text{poles} - \sum \text{zeros}}{n-m}$$

$$= \frac{(-1-2-3)-0}{3} = -2$$

- Loci on Re – axis: Loci lie on Re-axis in regions where there is an odd number of poles and zeros (on Re-axis) to the right of it.
- Intersection of RL with Im-axis will yield limits of gain for stability
  - Routh-Hurwitz
  - On Im-axis $s = jw$

  Substitute $s = jw$ in CE

  $CE: s^3 + 6s^2 + 11s + 6 + k = 0$

$$\Rightarrow (jw)^3 + 6(jw)^2 + 11(jw) + 6 + k = 0$$

$$\text{Re} \Rightarrow \left(-6\omega^2 + 6 + k\right) = 0$$

$$\text{Im} \Rightarrow \left(-\omega^3 + 11\omega\right)j = 0$$

$$\omega\left(11 - \omega^2\right) = 0 \Rightarrow \omega = 0$$

$$\omega = \pm\sqrt{11}$$

$$-6\left(\sqrt{11}\right)^2 + 6 + k = 0 \Rightarrow k = 60$$

- Breakin/Breakaway point

$$CE: 1 + kGH = 0 = f(s)$$

$$A + kB = 0 = f(s) \qquad \left(A = A(s),\ B = B(s)\right)$$

$$k = -\frac{A}{B}$$

$$\min/\max \Rightarrow \frac{dk}{ds} = -\frac{d}{ds}\left(\frac{A}{B}\right) = 0$$

The value of s is breakin/away pts if k

for that s is positive.