Quadratic Poles & Zeros

Quadratic Poles

$$G(s) = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$G(s) = \frac{1}{\omega_n^2} \frac{1}{\left(\frac{s}{\omega_n}\right)^2 + 2\zeta \frac{s}{\omega_n} + 1}$$

$$G(j\omega) = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + 2\zeta \frac{\omega}{\omega_n} j}$$

$$\text{Mag}: \quad 2 - \log\left|G(j\omega)\right| = -20\log\left|1 - \left(\frac{\omega}{\omega_n}\right)^2 + 2\zeta\frac{\omega}{\omega_n}j\right|$$

$$= -20 \times \frac{1}{2}\log\left[\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{2\zeta\frac{\omega}{\omega_n}\right\}^2\right]$$

$$\text{Let } \frac{\omega}{\omega_n} = u$$

$$= -10\log\left[\left(1 - u^2\right)^2 + \left(2\zeta u\right)^2\right]$$

$$\text{Phase} \quad \angle G(j\omega) = -\text{atan}\left(\frac{2\zeta u}{1-u^2}\right)$$

$$\text{if } u << 1, \text{ Mag} \approx -10\log 1 = 0$$

$$\text{Phase} \approx 0^o$$

$$\text{if } u >> 1, \text{ Mag} \approx -10\log u^4 = -40\log u$$

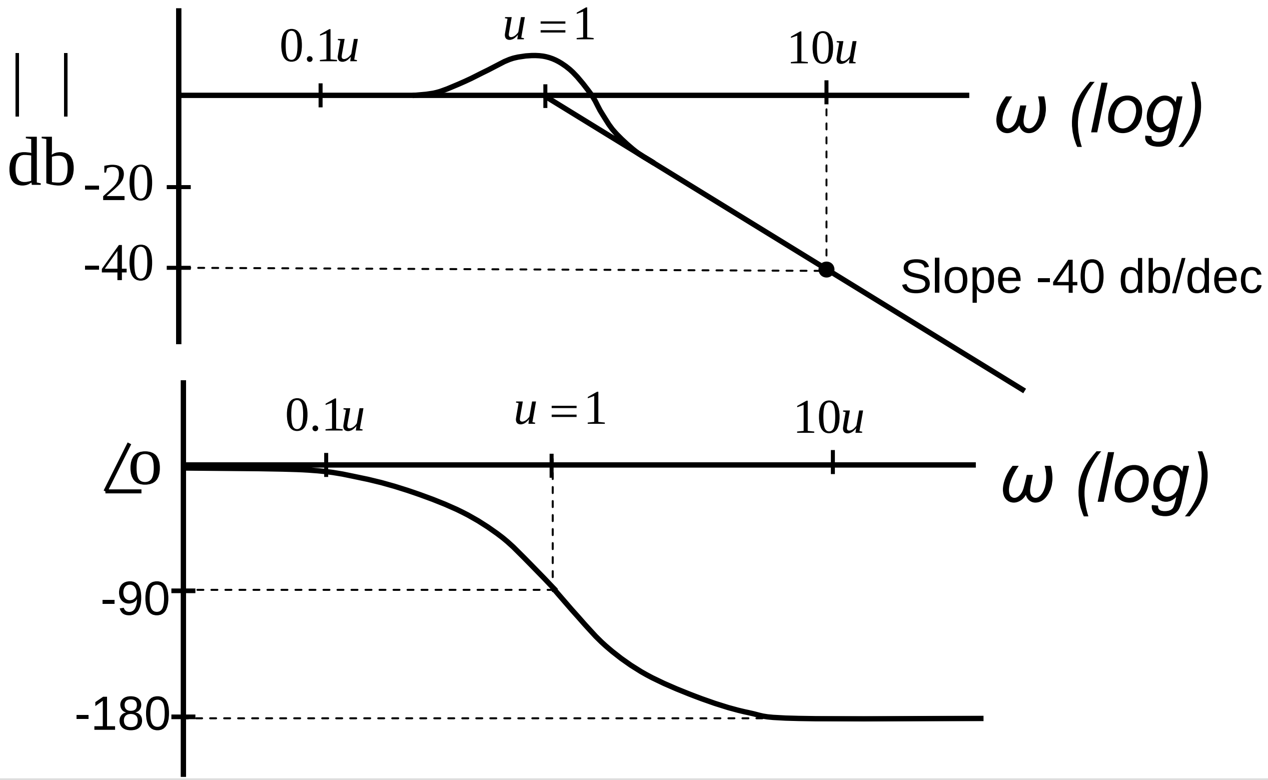
$$\text{slope} = -40\,\text{db/dec}$$

$$\text{Phase} \approx \text{-atan}\left(\frac{2\zeta u}{-u^2}\right) \approx -\text{atan}\left(\frac{1}{-u}\right)$$

$$\approx -180^o$$

$$\text{if } u = 1, \ \text{Mag} = -10\log(2\zeta)^2 = -20\log(2\zeta)$$

$$\text{Phase} = -\text{atan}\left(\frac{2\zeta}{0}\right) = -90^o$$



Max Amplitude occurs at ω_r (resonant frequency)

$$\omega_{\mathrm{r}} = \omega_n \sqrt{1 - 2\zeta^2} \qquad \zeta < 0.707$$

$$M_{P_{\omega r}} = \left| G(j\omega_{\mathrm{r}}) \right| = \left\{ 2\zeta\sqrt{1-\zeta^2} \right\}^{-1}$$

$$\text{Output} = \begin{pmatrix} \text{Mag} \\ \text{Input} \end{pmatrix} \times \left| G(j\omega) \right| \times \sin\left(\omega t + 0^o\right)$$

First Order Simple Pole

at $\omega = {}^{0.1}\!/\!_{\tau} \Rightarrow 0 \text{ db}$

$$20\log\left|G(j\omega)\right| = 0 \text{ db} \Rightarrow \left|G(j\omega)\right| = 1$$

$$\text{output} = \begin{pmatrix}\text{mag} \\ \text{input}\end{pmatrix} \times 1 \times \sin(\omega t + 0^{o})$$

at $\omega = {}^{100}\!/\!_{\tau}$

$$20\log\left|G(j\omega)\right| = -40 \text{ db} \Rightarrow \left|G(j\omega)\right| = 0.01$$

$$\angle \approx -90^{o} \quad \text{output} = \begin{pmatrix}\text{mag} \\ \text{input}\end{pmatrix} \times 0.01 \times \sin(\omega t - 90^{o})$$

Example

$$G(s) = \frac{10\,(s+3)}{s\ (s+2)(s^2+s+2)}$$

Normalize

$$G(s) = \frac{10 \times 3\left(\frac{1}{3}s+1\right)}{s \times 2\left(\frac{1}{2}s+1\right) \times 2\left(\left(\frac{s}{\sqrt{2}}\right)^2 + \frac{1}{2}s+1\right)}$$

$$G(j\omega) = \frac{7.5\left(\frac{j\omega}{3}+1\right)}{j\omega\left(\frac{j\omega}{2}+1\right)\left(\left(\frac{j\omega}{\sqrt{2}}\right)^2 + \frac{j\omega}{2}+1\right)}$$

FACTORS		7.5	$j\frac{\omega}{3}+1$	$(j\omega)^{-1}$	$\left(j\frac{\omega}{2}+1\right)^{-1}$	$(Quad)^{-1}$
Break Freq.		*	3	1	2	$\sqrt{2}$
Slopes db/dec	Low Freq.	*	0	-20	0	0
	High Freq.	*	20	-20	-20	-40
Angle deg.	Low Freq.	0	0	-90	0	0
	Break Freq.		45		-45	-90
	High Freq.	0	90	-90	-90	-180

$$\text{Final Slope}\,(\omega >> 1) = -60\,\text{db/dec}$$

$$\text{Final Phase}\,(\omega >> 1) = -270^{o}$$

Constant :

$$20\log(\text{Constant})\,\text{db} =$$

$$20\log(7.5) = 17.5\,\text{db}$$

Quadratic :

Find $\omega_r, \zeta, M_{p\omega_r}$ from formulae.