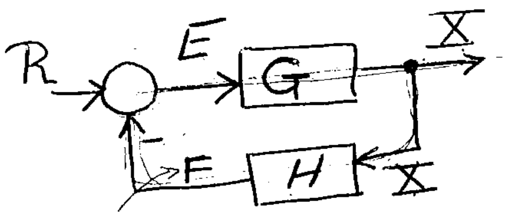
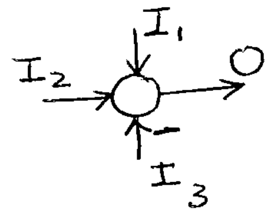
open loop system

$$X(s) = G(s)\,F(s)$$

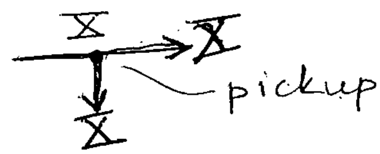
closed loop system





$$\sum I = 0$$

$$I_1 + I_2 \overset{-}{\oplus} I_3 = 0$$



$$X = f(R, G, H)$$

$$\left.\begin{array}{l} X = EG \\ \left.\begin{array}{l} F = XH \\ E = R - F \end{array}\right\} E = R - HX \end{array}\right\} X = (R - HX)G$$

$$X = RG - HGX$$

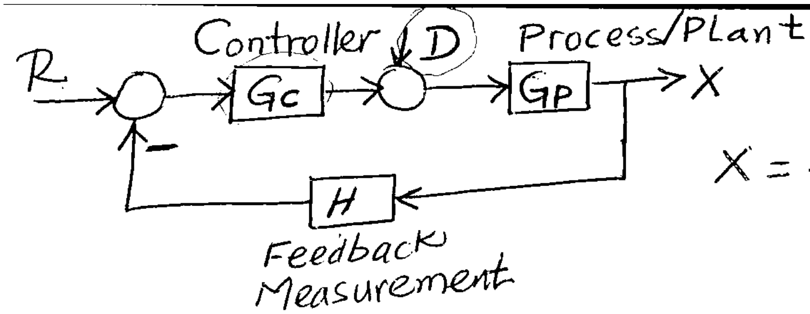
$$X + GHX = RG$$

$$X(1 + GH) = RG \Rightarrow \frac{X}{R} = \frac{G}{1 + GH}$$

closed loop transfer function c.l.t.f.

$$\frac{X}{R} = \frac{\text{Feed Forward Path}}{1 + (F.F.) * (F.B)}$$

negative feedback



$X = f(R, G_c, G_p, D)$

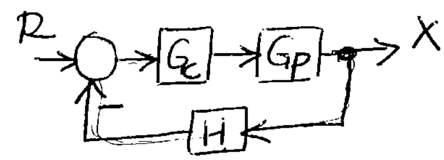
LTI Linear Time Invariant

Superposition $X = f(\quad)$

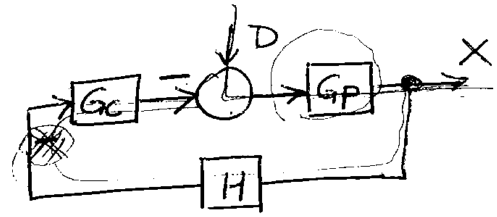
$$X = X_R\Big|_{\text{all other inputs}=0} + X_D\Big|_{\text{all other inputs}=0}$$

$$X_R\Big|_{D=0}$$

$$X_R\Big|_{D=0} = \frac{G_c G_p}{1 + G_c G_p H} R$$



$$X_D\Big|_{R=0} = \frac{G_p}{1 + G_p H G_c} D$$



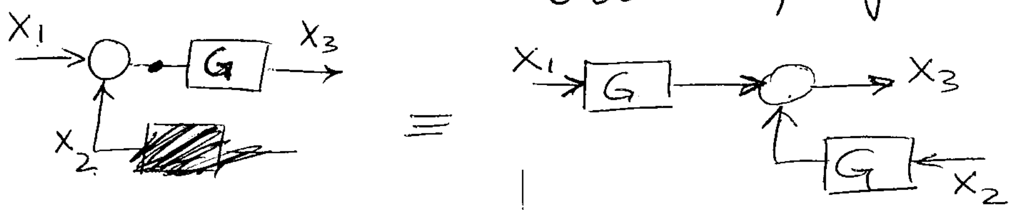
$$X = X_R\Big|_{D=0} + X_D\Big|_{R=0}$$

$$X = \frac{G_c G_p}{1 + G_c G_p H} R + \frac{G_p}{1 + G_c G_p H} D$$

$1 + G_c G_p H$ characteristic polynomia CP

$1 + G_c G_p H = 0$ —"— equation Ci

$1 + G_c G_p H = 0$ CE $\Longrightarrow$ eigenvalues
stability
nature of response
order of system



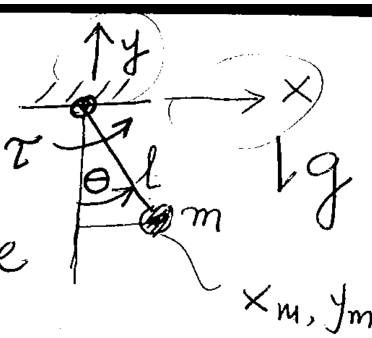
$(x_1 + x_2) G = X_3$

$X_1 G + X_2 G = X_3$

$X_3 = G X_1 + G X_2$

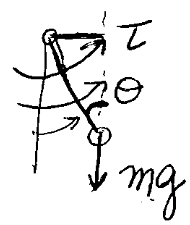
Modeling Review

Pendulum



- Need x, y to define the position of mass.
- $\tan\theta = \dfrac{x_m}{y_m}$; $\sqrt{x_m^2 + y_m^2} = l$

$$\Sigma T_o = I_o \theta''$$



$\sin\theta = \dfrac{x_m}{l}$

$x_m = l s\theta$

$$\tau - mgl\sin\theta = ml^2\theta''$$

Non Linear System

Linearization

Equilibrium - **Static** - position

$\tau = 0$, $\ddot{\theta} \Rightarrow (mgl)\sin\theta = 0$

$\sin\theta = 0 \to 0^\circ$, 180°

if τ = value - equilibrium pt

$\sin\theta = \frac{\tau}{mgl}$

Linearize with Taylor Series.

$f = ml^2\ddot{\theta} + mgl\sin\theta - \tau = 0$

$f(m, l, g, \theta, \ddot{\theta}, \tau)$ $\quad \Delta m \to 0 \quad \left.\frac{\partial f}{\partial m}\right|_0$

$$\underbrace{f - f_0}_{\Delta f} = \left(\frac{\partial f}{\partial m}\right)_0 \cancel{(m - m_0)} + \left.\frac{\partial f}{\partial l}\right|_0 \cancel{(l - l_0)} + \left.\frac{\partial f}{\partial g}\right|_0 \cancel{(g - g_0)} + \left.\frac{\partial f}{\partial \theta}\right|_0 (\theta - \theta_0) + \left.\frac{\partial f}{\partial \ddot{\theta}}\right|_0 (\ddot{\theta} - \ddot{\theta}_0) + \left.\frac{\partial f}{\partial \tau}\right|_0 (\tau - \tau_0)$$

Δ

$$\left(s^2 + g/l\right) \Delta\Theta = \frac{1}{ml^2}\Delta T$$

$$\frac{\Delta\Theta}{\Delta T} = \frac{1}{ml^2\left(s^2+g/l\right)}$$

CE: $ml^2\left(s^2+g/l\right)=0$

$$s^2+g/l=0$$

Eigenvalues - Roots of CE

$$s^2=-g/l \Rightarrow s_{1,2}=\pm\sqrt{-g/l}=\pm\sqrt{g/l}\cdot j$$