Sensitivity Analysis

Open Loop | **Closed Loop**

Open Loop

H=0

$$C = G_p T + G_c G_p R$$

Closed Loop

$$C = \frac{G_p}{1 + G_c G_p H} T + \frac{G_c G_p}{1 + G_c G_p H} R$$

1. Sensitivity to disturbances: $\Delta R = 0 \quad \Delta T \neq 0$

Open Loop

$$\Delta C = G_p \Delta T$$

Disturbances propagates to the output with gain G_p. **If** $|G_p| > 1$, then the disturbance is amplified.

Closed Loop

$$\Delta C = \frac{G_p}{1 + G_c G_p H} \Delta T$$

As the closed loop gain $G_c G_p H$ increases, and **If** $\|G_c G_p H\| » 1$, then $\Delta C \cong \frac{1}{G_c H} \Delta T$ indicating that the distrubance is attenuated

2. Sensitivity to plant parameter changes: $T=0 \quad R \neq 0 \quad \Delta G_p \neq 0$

Open Loop

$$\Delta C = G_c R \Delta G_p$$

Normalizing $\frac{\Delta C}{C}$

$$\frac{\Delta C}{C} = \frac{G_c R}{G_c G_p R} \Delta G_p = \frac{1}{G_p} \Delta G_p$$

$$\therefore \frac{\Delta C}{C} = 1 \frac{\Delta G_p}{G_p}$$

One to one mapping

Closed Loop

$$\Delta C = \frac{G_c R}{(1 + G_c G_p H)^2} \Delta G_p$$

Normalizing $\frac{\Delta C}{C}$

$$\frac{\Delta C}{C} = \left(\frac{1}{1 + G_c G_p H}\right) \frac{\Delta G_p}{G_p}$$

As $G_c G_p H$ increases, then $\frac{\Delta C}{C}$ decreases.

Closed Loop feedback reduces the change on the output if the plant parameters change, i.e. this effect is attenuated on the output.

If $\|G_c G_p H\| » 1$, then $\frac{\Delta C}{C} \cong \frac{1}{(G_c G_p H)} \frac{\Delta G_p}{G_p}$

indicating that this effect is attenuated

3. Sensitivity to controller gain changes: $T = 0 \;\; R \neq 0$

$$\Delta C = G_p R \Delta G_c$$

Normalizing $\frac{\Delta C}{C}$

$$\frac{\Delta C}{C} = \frac{\Delta G_c}{G_c}$$

One to one mapping

$$\Delta C = \frac{G_p R}{(1 + G_c G_p H)^2} \Delta G_c$$

Normalizing $\frac{\Delta C}{C}$

$$\frac{\Delta C}{C} = \frac{1}{(1 + G_c G_p H)} \frac{\Delta G_c}{G_c}$$

As $G_c G_p H$ increases, then $\frac{\Delta C}{C}$ decreases.

Closed loop feedback reduces the sensitivity of the output with respect to changes in the controller gain. This effect is attenuated on the output.

If $\|G_c G_p H\| \gg 1$, then $\frac{\Delta C}{C} \cong \frac{1}{(G_c G_p H)} \frac{\Delta G_c}{G_c}$

indicating that this effect is attenuated.

4. Sensitivity to feedback gain: $H \neq 0$

No sensitivity since there is no feedback!

$$\Delta C = \frac{-G_c^2 G_p^2 R}{(1 + G_c G_p H)^2} \Delta H$$

Normalizing $\frac{\Delta C}{C}$

$$\frac{\Delta C}{C} = \frac{-G_c G_p}{1 + G_c G_p H} \Delta H$$

if $\|G_c G_p H\| \gg 1$, then $\frac{\Delta C}{C} \cong -\frac{\Delta H}{H}$

One to one mapping

Note: H is what is measured from a sensor; therefore an accurate and good sensor should be used since there is an one-to-one mapping with the output.

